# *Imperative Programming with The State Monad*

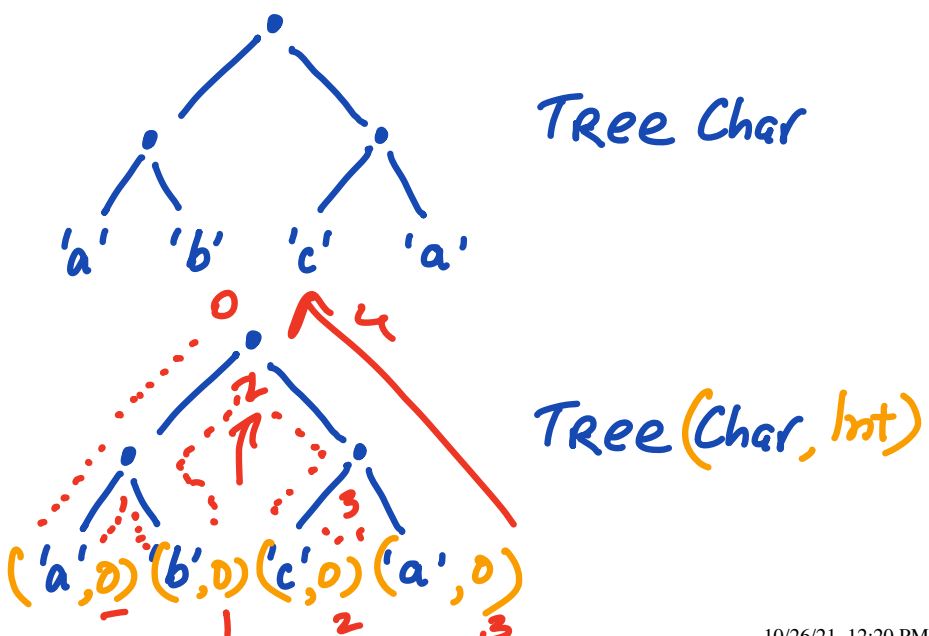
# *A Tree Datatype*

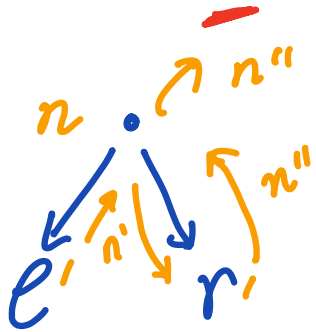
A tree with data at the **leaves**

```
data Tree a
    = Leaf a
    | Node (Tree a) (Tree a)
    deriving (Eq, Show)
```

Here's an example `Tree Char`

```
charT :: Tree Char
charT = Node
            (Node
                (Leaf 'a')
                (Leaf 'b'))
            (Node
                (Leaf 'c')
                (Leaf 'a'))
```






# *Lets Work it Out!*

Write a function to add a *distinct* label to each *leaf*

```
label :: Tree a -> Tree (a, Int)
label = ???
```

such that

```
>>> label charT
Node
    (Node
        (Leaf ('a', 0))
        (Leaf ('b', 1)))
    (Node
        (Leaf ('c', 2))
        (Leaf ('a', 3)))
```

# *Labeling a Tree*

```
label :: Tree a -> Tree (a, Int)
label t       = t'
  where
      (_, t') = (helper 0 t)

helper :: Int -> (Int, Tree (a, Int))
helper n (Leaf x)   = (n+1, Leaf (x, n))
helper n (Node l r) = (n'', Node l' r')
  where
      (n', l')      = helper n l
      (n'', r')     = helper n' r
```

# *EXERCISE*

Now, modify `label` so that you get new numbers for each `letter` so,

```
>>> keyLabel (Node (Node (Leaf 'a') (Leaf 'b')) (Node (Leaf 'c') (Lea
f 'a')))
    (Node
        (Node (Leaf ('a', 0)) (Leaf ('b', 0)))
        (Node (Leaf ('c', 0)) (Leaf ('a', 1))))
```

That is, a *separate* counter for each *key* `a` , `b` , `c` etc.

**HINT** Use the following `Map k v` type

```
-- | The empty Map
empty :: Map k v

-- | 'insert key val m` returns a new map that extends 'm'
--   by setting `key` to `val`
insert :: k -> v -> Map k v -> Map k v

-- | 'findWithDefault def key m' returns the value of `key`
--   in `m`  or `def` if `key` is not defined
findWithDefault :: v -> k -> Map k v -> v
```

# *Common Pattern?*

Both the functions have a common "shape"

```
OldInt -> (NewInt, NewTree)
```

```
OldMap -> (NewMap, NewTree)
```

If we generally think of `Int` and `Map Char Int` as **global state**

```
OldState -> (NewState, NewVal)
```

# *State Transformers*

Lets capture the above "pattern" as a type

1. A **State** Type

```
type State = ... -- lets "fix" it to Int for now...
```

2. A **State Transformer** Type

```
data ST a = STC (State -> (State, a))
```

A *state transformer* is a function that

- takes as input an **old** `s :: State`
- returns as output a **new** `s' :: State` and **value** `v :: a`

# *Executing Transformers*

Lets write a function to *evaluate* an `ST a`

```
evalState :: State -> ST a -> a
evalState = ???
```

# *QUIZ*

What is the value of `quiz` ?

```
st :: St [Int]
st = STC (\n -> (n+3, [n, n+1, n+2]))

quiz = evalState 100 st
```

**A.** `103`

**B.** `[100, 101, 102]`

**C.** `(103, [100, 101, 102])`

**D.** `[0, 1, 2]`

**E.** Type error

# *Lets Make State Transformer a Monad!*

```
instance Monad ST where
    return :: a -> ST a
    return = returnST

    (>>=)  :: ST a -> (a -> ST b) -> ST b
    (>>=) = bindST
```

# *EXERCISE: Implement* `returnST`*!*

What is a valid implementation of `returnST` ?

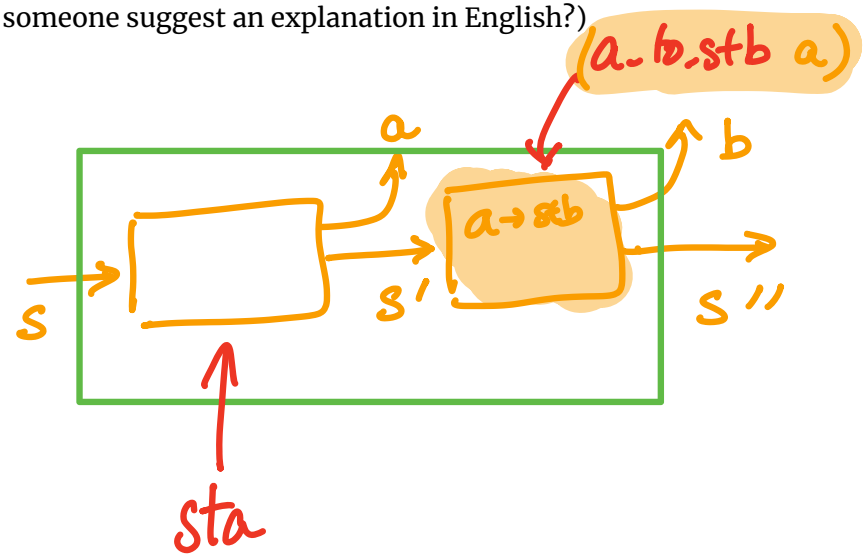
```
type State = Int
data ST a  = STC (State -> (State, a))

returnST :: a -> ST a
returnST = ???
```

# *What is* `returnST` *doing ?*

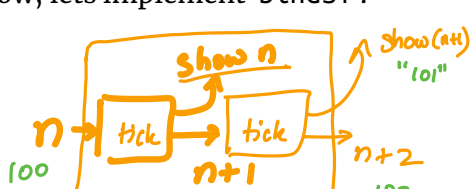
`returnST v` is a *state transformer* that ... ???

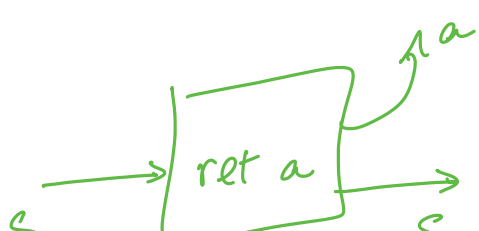
(Can someone suggest an explanation in English?)




# *HELP*

Now, lets implement `bindST` !

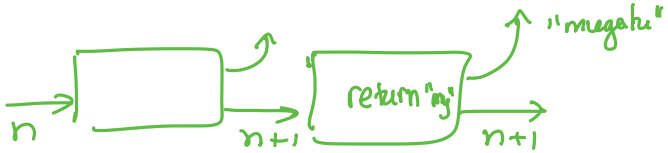


```
type State = Int

data ST a  = STC (State -> (State, a))

bindST :: ST a -> (a -> ST b) -> ST b
bindST = ???
```




# *What is `bindST` doing?*

`bindST v` is a *state transformer* that ... ???

(Can someone suggest an explanation in English?)

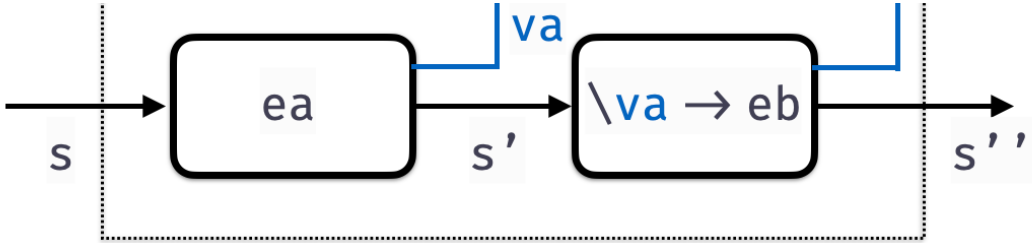
*`bindST` lets us **sequence** state transformers*

```
(>>=) :: ST0 a -> (a -> ST0 b) -> ST0 b
sta >>= f = STC (\s ->
                    let (s', va)  = runState sta s
                        stb       = f va
                        (s'', vb) = runState stb s'
                    in
                        (s'', vb)
                )
```

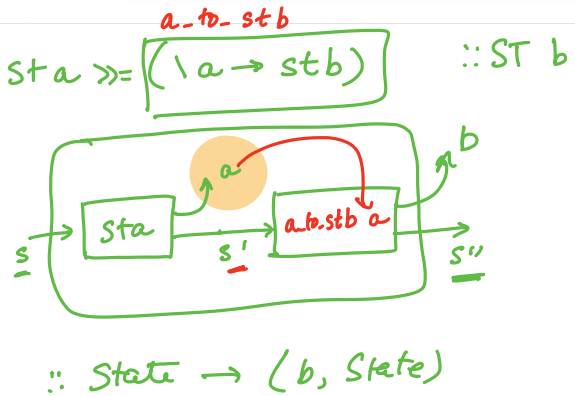
```
st >>= f
```

1. Applies transformer `st` to an initial state `s`
    - to get output `s'` and value `va`
2. Then applies function `f` to the resulting value `va`
    - to get a *second* transformer
3. The *second* transformer is applied to `s'`
    - to get final `s''` and value `vb`

**OVERALL:** Transform `s` to `s''` and produce value `vb`




```
ea >>= (\va → eb)
```




# *Lets Implement a Global Counter*

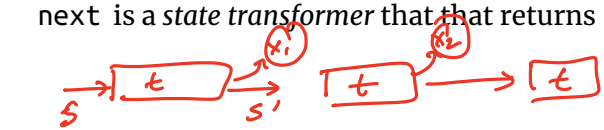
The (counter) `State` is an `Int`

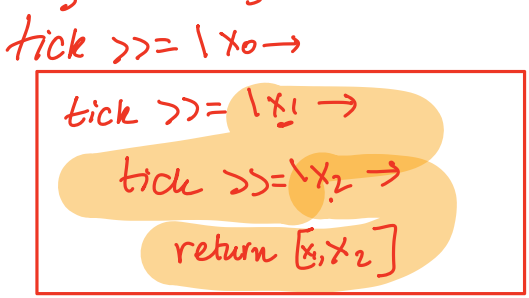
```
type State = Int
```

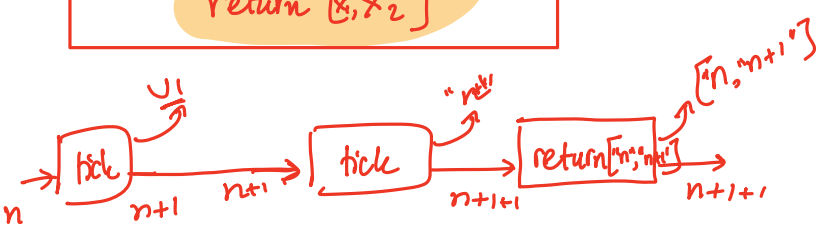
A function that *increments* the counter to *return* the `next Int`.

```
next :: ST String
next = STC (\s -> (s+1, show s))
```

`next` is a *state transformer* that that returns `String` values




# *QUIZ*

Recall that

```
evalState :: State -> ST a -> a
evalState s (STC st) = snd (st s)

next :: ST String
next = STC (\s -> (s+1, show s))
```

What does `quiz` evaluate to?

```
quiz = evalState 100 next
```

**A.** `"100"`

**B.** `"101"`

**C.** `"0"`

**D.** `"1"`

**E.** `(101, "100")`

# *QUIZ*

Recall the definitions

```
evalState :: State -> ST a -> a
evalState s (STC st) = snd (st s)

next :: ST String
next = STC (\s -> (s+1, show s))
```

Now suppose we have

```
wtf1 = ST String
wtf1 = next >>= \n ->
         return n
```

What does `quiz` evaluate to?

```
quiz = evalState 100 wtf1
```

**A.** `"100"`

**B.** `"101"`

**C.** `"0"`

**D.** `"1"`

**E.** `(101, "100")`

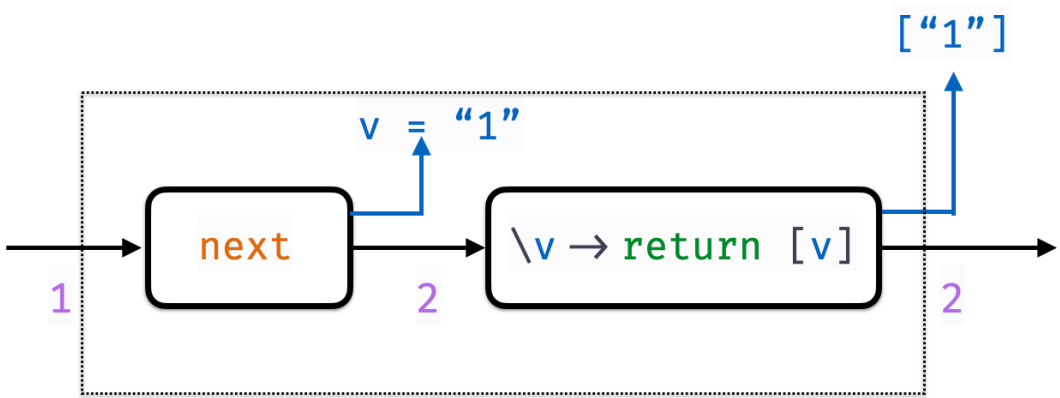
# *Example*

```
next :: ST0 String
```

```haskell
next = ST0C (\s → (s+1, show s))

wtf :: ST0 [String]
wtf  = next >>= (\v → return [v])

quiz = evalState wtf 1
```

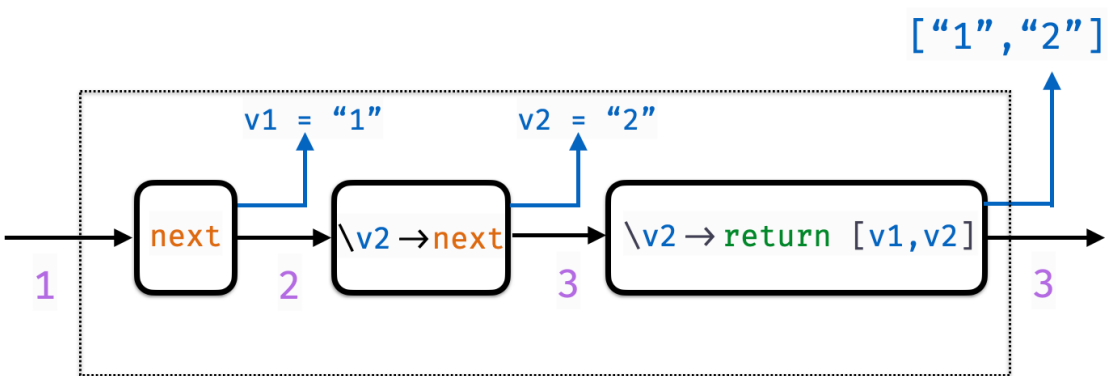


# *Example*

```
next :: ST0 String
next = ST0C (\s → (s+1, show s))

wtf :: ST0 [String]
wtf  = next >>= (\v1 → next >>= (\v2 → return [v1, v2]))

quiz = evalState wtf 1
```



# *QUIZ*

```
next :: ST String
next = STC (\s -> (s+1, show s)

evalState :: State -> ST a -> a
evalState s (STC f) = snd (f s)
```

Consider a function `wtf2` defined as

```
wtf2 = next >>= \n1 ->
         next >>= \n2 ->
           next >>= \n3 ->
             return [n1, n2, n3]
```

What does `quiz` evaluate to?

```
quiz = evalState 100 wtf
```

**A.** Type Error!

**B.** [“100”, “100”, “100”]

**C.** [“0”, “0”, “0”]

**D.** [“100”, “101”, “102”]

**E.** [“102”, “102”, “102”]

# *Chaining Transformers*

`>>=` lets us *chain* transformers into *one* big transformer!

So we can define a function to *increment the counter by 3*

```
-- Increment the counter by 3
next3 :: ST [Int]
next3 = next >>= \n1 ->
          next >>= \n2 ->
            next >>= \n3 ->
                return [n1,n2,n3]
```

And then sequence it *twice* to get

```
next6 :: ST [Int]
next6 = next3 >>= \ns_1_2_3 ->
          next3 >>= \ns_4_5_6 ->
            return (ns_123 ++ ns_4_5_6)
```

# *Lets* **do** *the above examples*

Remember, **do** is just nice syntax for the above!

```
-- Increment the counter by 3
next3 :: ST [Int, Int]
next3 = do
  n1 <- next
  n2 <- next
  n3 <- next
  return [n1,n2,n3]
```

And then sequence it *twice* to get

```
next6 :: ST [Int]
next6 = do
  ns_123 <- next3
  ns_456 <- next3
  return (ns_123 ++ ns_4_5_6)
```

# *Labeling a Tree with a "Global Counter"*

Lets **rewrite** our `Tree` labeler with `ST`

```
helperS :: Tree a -> ST (Tree (a, Int))
helperS = ???
```

## *Wow, compare to the old code!*

```
helper :: Int -> (Int, Tree (a, Int))
helper n (Leaf x)   = (n+1, Leaf (x, n))
helper n (Node l r) = (n'', Node l' r')
  where
      (n', l')      = helper n l
      (n'', r')     = helper n' r
```

Avoid worrying about propagating the "right" counters

- Automatically handled by `ST` monad instance!

# *Executing the Transformer*

In the **old** code we *called* the helper with an *initial* counter `0`

```
label :: Tree a -> Tree (a, Int)
label t       = t'
  where
      (_, t') = helper 0 t
```

In the **new** code what should we do?

```
helperS :: Tree a -> ST (Tree (a, Int))
helperS = ...

labelS :: Tree a -> Tree (a, Int)
labelS = ???
```

Now, we should be able to `exec` the `labelS` transformer

```
>>> labelS (Node (Node (Leaf 'a') (Leaf 'b')) (Leaf 'c'))
(Node (Node (Leaf ('a', 0)) (Leaf ('b', 1))) (Leaf ('c', 2)))
```

# *How to implement* `keyLabel`*?*

So far, we *hardwired* an `Int` counter as our `State`

```
type State = Int

data ST a  = STC (State -> (State, a))
```

Have to *reimplement* the monad if we want a *different* state?

- e.g. `Map Char Int` to implement `keyLabel`