# *Imperative Programming with The State Monad*

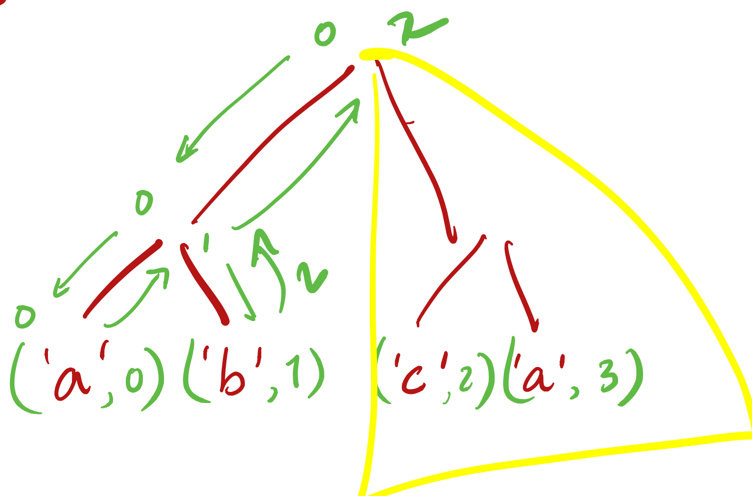
## *A Tree Datatype*

A tree with data at the **leaves**

```
data Tree a
    = Leaf a
    | Node (Tree a) (Tree a)
    deriving (Eq, Show)
```

Here's an example `Tree Char`

```
charT :: Tree Char
charT = Node
          (Node
              (Leaf 'a')
              (Leaf 'b'))
          (Node
              (Leaf 'c')
              (Leaf 'a'))
```



# *Lets Work it Out!*

Write a function to add a *distinct* label to each *leaf*

```
label :: Tree a -> Tree (a, Int)
label = ???
```

such that

```
>>> label charT
Node
    (Node
        (Leaf ('a', 0))
        (Leaf ('b', 1)))
    (Node
        (Leaf ('c', 2))
        (Leaf ('a', 3)))
```

# *Labeling a Tree*

```haskell
label :: Tree a -> Tree (a, Int)
label t       = t'
  where
      (_, t') = (helper 0 t)

helper :: Int -> (Int, Tree (a, Int))
helper n (Leaf x)   = (n+1, Leaf (x, n))
helper n (Node l r) = (n'', Node l' r')
  where
      (n', l')      = helper n l
      (n'', r')     = helper n' r
```

# *EXERCISE*

Now, modify `label` so that you get new numbers for each `letter` so,

```
>>> keyLabel (Node (Node (Leaf 'a') (Leaf 'b')) (Node (Leaf 'c') (Leaf 'a')))
    (Node
        (Node (Leaf ('a', 0)) (Leaf ('b', 0)))
        (Node (Leaf ('c', 0)) (Leaf ('a', 1))))
```

That is, a *separate* counter for each *key* `a`, `b`, `c` etc.

**HINT** Use the following `Map k v` type

```
-- | The empty Map
empty :: Map k v

-- | 'insert key val m` returns a new map that extends 'm'
--   by setting `key` to `val`
insert :: k -> v -> Map k v -> Map k v

-- | 'findWithDefault def key m' returns the value of `key`
--   in `m`  or `def` if `key` is not defined
findWithDefault :: v -> k -> Map k v -> v
```

# *Common Pattern?*

Both the functions have a common "shape"

```
OldInt -> (NewInt, NewTree)

OldMap -> (NewMap, NewTree)
```

If we generally think of `Int` and `Map Char Int` as **global state**

```
OldState -> (NewState, NewVal)
```

# *State Transformers*

Lets capture the above "pattern" as a type

1. A **State** Type

```
type State = ... -- lets "fix" it to Int for now...
```

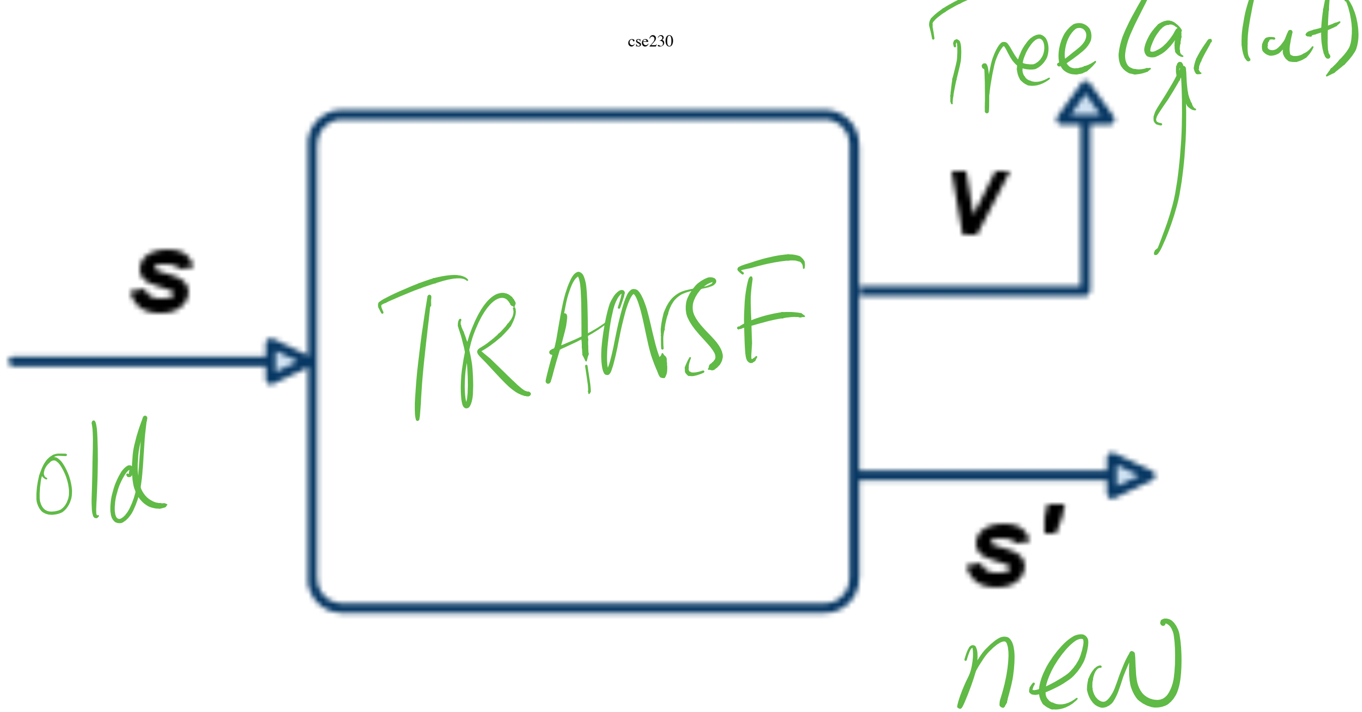
2. A **State Transformer** Type

OLD NEW VAL

```
data ST a = STC (State -> (State, a))
```

A *state transformer* is a function that

- takes as input an **old** `s :: State`
- returns as output a **new** `s' :: State` and **value** `v :: a`


Tree (a, Int)
TRANSF
S
old
V
S'
new

# *Executing Transformers*

Lets write a function to *evaluate* an `ST a`

```
evalState:: State -> ST a -> a
evalState= ???
```

# *QUIZ*

What is the value of `quiz` ?

```
st :: St [Int]
st = STC (\n -> (n+3, [n, n+1, n+2]))

quiz = evalState100 st
```

**A.** `103`

**B.** `[100, 101, 102]`

**C.** `(103, [100, 101, 102])`

**D.** `[0, 1, 2]`

**E.** Type error

# *Lets Make State Transformer a Monad!*

```
instance Monad ST where
    return :: a -> ST a
    return = returnST

    (>>=)  :: ST a -> (a -> ST b) -> ST b
    (>>=) = bindST
```

# *EXERCISE: Implement* `returnST`*!*

What is a valid implementation of `returnST`?

```haskell
type State = Int
data ST a  = STC (State -> (State, a))

returnST :: a -> ST a
returnST = ???
```

# *What is `returnST` doing?*

`returnST v` is a *state transformer* that … ???

(Can someone suggest an explanation in English?)

# *HELP*

Now, lets implement `bindST`!

```
type State = Int

data ST a  = STC (State -> (State, a))

bindST :: ST a -> (a -> ST b) -> ST b
bindST = ???
```

# *What is `returnST` doing ?*

`returnST v` is a *state transformer* that ... ???

(Can someone suggest an explanation in English?)

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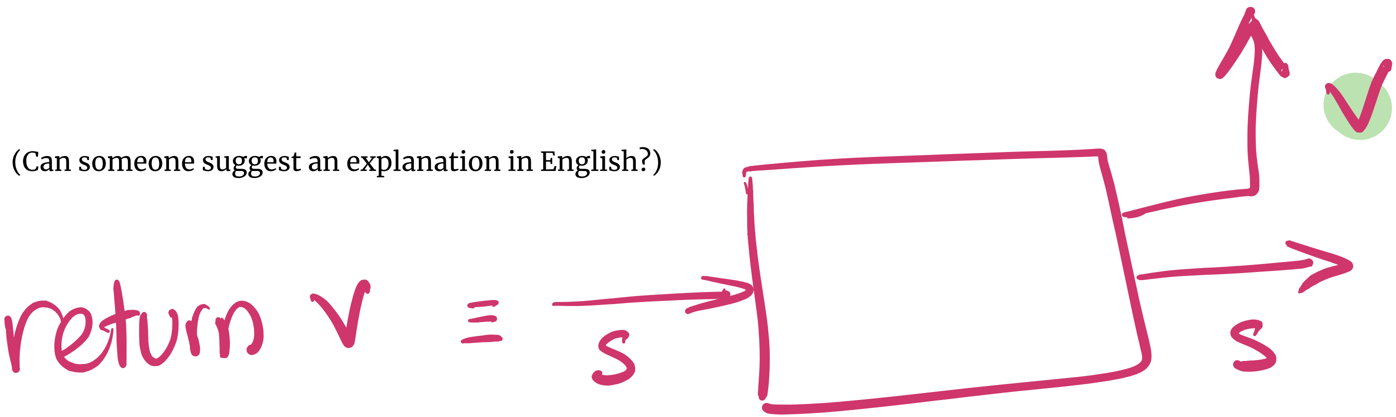
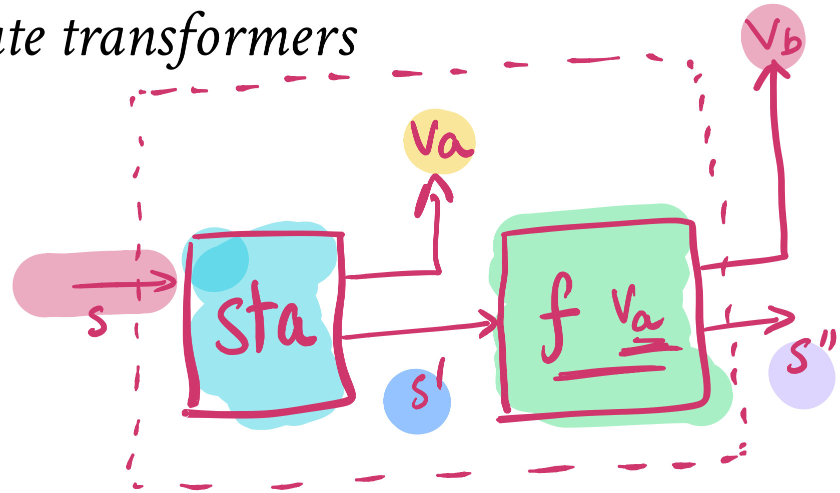


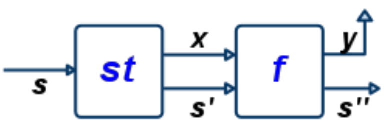
# `bindST` *lets us **sequence** state transformers*

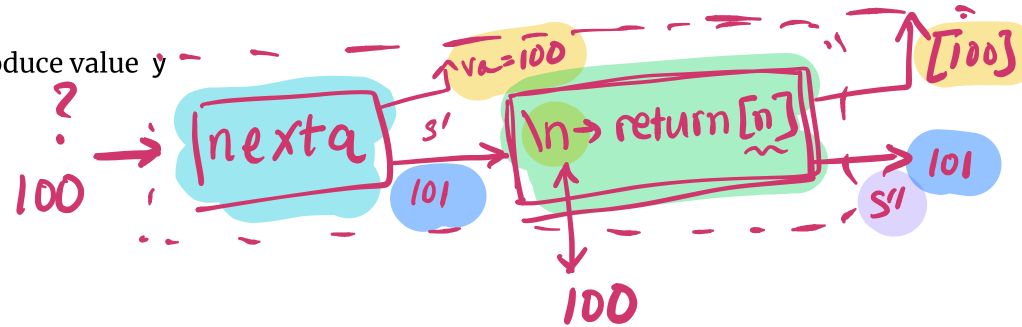


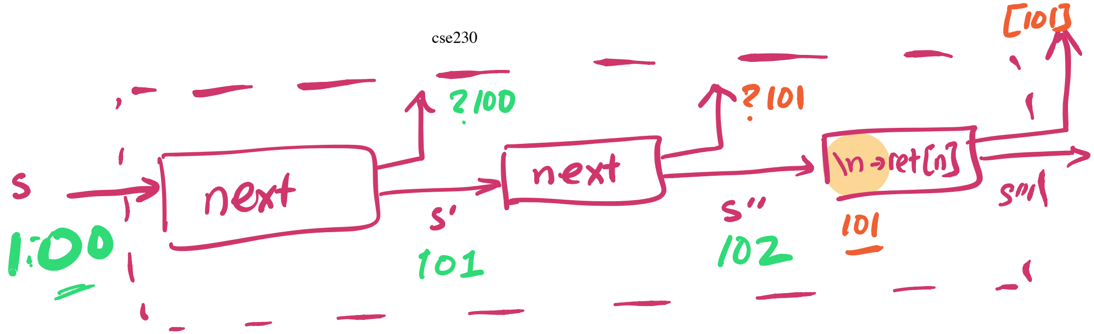

```
st >>= f
```

1. Applies transformer `st` to an initial state `s`
    - to get output `s'` and value `x`
2. Then applies function `f` to the resulting value `x`
    - to get a *second* transformer
3. The *second* transformer is applied to `s'`
    - to get final `s''` and value `y`

**OVERALL:** Transform `s` to `s''` and produce value `y`






# *Lets Implement a Global Counter*

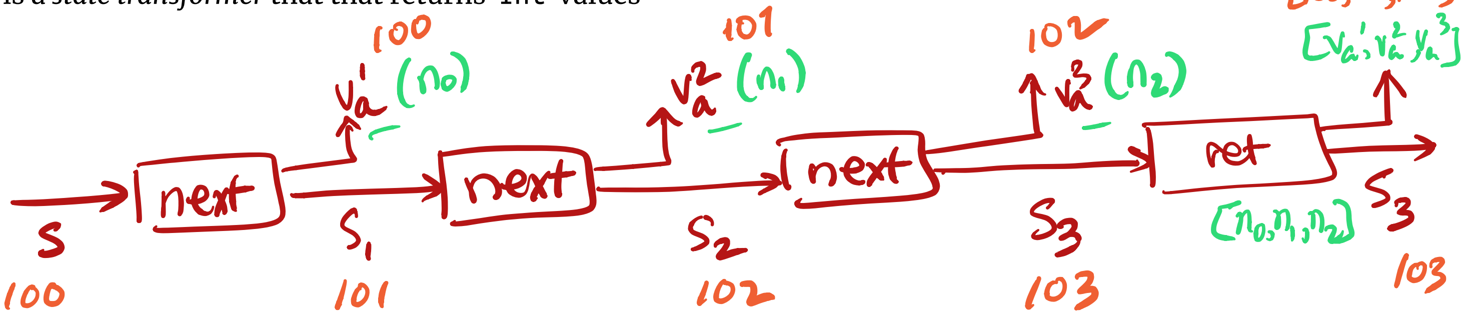
The (counter) `State` is an `Int`

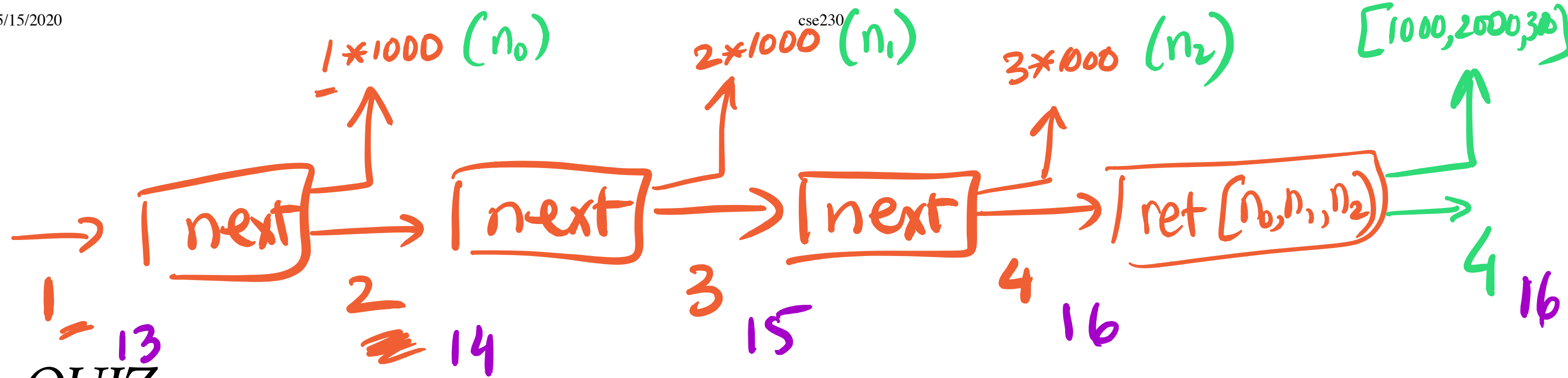
```
type State = Int
```

A function that *increments* the counter to *return* the `next Int`.

```
next :: ST Int
next = STC (\old -> let new = old + 1 in (new, old))
```

`next` is a *state transformer* that that returns `Int` values






# *QUIZ*

Recall that

```
evalState :: State -> ST a -> a
evalState s (STC st) = snd (st s)
```

```
next :: ST Int
next = STC (\n -> (n+1, n))
```

What does `quiz` evaluate to?

```
quiz = evalState 100 next
```

**A.** `100`

**B.** `101`

**C.** `0`

**D.** `1`

**E.** `(101, 100)`

# *QUIZ*

Recall the definitions

```
evalState :: State -> ST a -> a
evalState s (STC st) = snd (st s)

next :: ST Int
next = STC (\n -> (n+1, n))
```

Now suppose we have

```
wtf1 = ST Int
wtf1 = next >>= \n ->
         return n
```

What does `quiz` evaluate to?

```
quiz = evalState 100 wtf1
```

**A.** `100`

**B.** `101`

**C.** `0`

**D.** `1`

**E.** `(101, 100)`

# *QUIZ*

Consider a function `wtf2` defined as

```
wtf2 = next >>= \n1 ->
         next >>= \n2 ->
           next >>= \n3 ->
             return [n1, n2, n3]
```

What does `quiz` evaluate to?

```
quiz = evalState 100 wtf
```

**A.** Type Error!

**B.** [100, 100, 100]

**C.** [0, 0, 0]

**D.** [100, 101, 102]

**E.** [102, 102, 102]

# *Chaining Transformers*

`>>=` lets us *chain* transformers into *one* big transformer!

So we can define a function to *increment the counter by 3*

```
-- Increment the counter by 3
next3 :: ST [Int, Int]
next3 = next >>= \n1 ->
          next >>= \n2 ->
            next >>= \n3 ->
                return [n1,n2,n3]
```

And then sequence it *twice* to get

```
next6 :: ST [Int]
next6 = next3 >>= \ns_1_2_3 ->
          next3 >>= \ns_4_5_6 ->
            return (ns_123 ++ ns_4_5_6)
```

# *Lets* **do** *the above examples*

Remember, **do** is just nice syntax for the above!

```
-- Increment the counter by 3
next3 :: ST [Int, Int]
next3 = do
  n1 <- next
  n2 <- next
  n3 <- next
  return [n1,n2,n3]
```

And then sequence it *twice* to get

```
next6 :: ST [Int]
next6 = do
  ns_123 <- next3
  ns_456 <- next3
  return (ns_123 ++ ns_4_5_6)
```

# *Labeling a Tree with a "Global Counter"*

Lets **rewrite** our `Tree` labeler with `ST`

```
helperS :: Tree a -> ST (Tree (a, Int))
helperS = ???
```

## *Wow, compare to the old code!*

```
helper :: Int -> (Int, Tree (a, Int))
helper n (Leaf x)   = (n+1, Leaf (x, n))
helper n (Node l r) = (n'', Node l' r')
  where
      (n', l')      = helper n l
      (n'', r')     = helper n' r
```

Avoid worrying about propagating the "right" counters

- Automatically handled by `ST` monad instance!

# *Executing the Transformer*

In the **old** code we *called* the helper with an *initial* counter `0`

```
label :: Tree a -> Tree (a, Int)
label t       = t'
  where
      (_, t') = helper 0 t
```

In the **new** code what should we do?

```
helperS :: Tree a -> ST (Tree (a, Int))
helperS = ...

labelS :: Tree a -> Tree (a, Int)
labelS = ???
```

Now, we should be able to `exec` the `labelS` transformer

```
>>> labelS (Node (Node (Leaf 'a') (Leaf 'b')) (Leaf 'c'))
(Node (Node (Leaf ('a', 0)) (Leaf ('b', 1))) (Leaf ('c', 2)))
```

# *How to implement* `keyLabel`*?*

So far, we *hardwired* an `Int` counter as our `State`

```
type State = Int

data ST a  = STC (State -> (State, a))
```

Have to *reimplement* the monad if we want a *different* state?

- e.g. `Map Char Int` to implement `keyLabel`

**Don't Repeat Yourself!**

# *A Generic State Transformer*

Don't have *separate* types for `IntList` and `CharList`

- Define a generic list `[a]` where `a` is a *type parameter*
- Instantiate `a` to get `[Int]` and `[Char]`

Similarly, reuse `ST` with a **type** parameter!

```
data ST s a = STC (s -> (s, a))
```

- **State** is represented by type `s`
- **Return Value** is the type `a` (as before).

# *A Generic State Transformer Monad*

Lets make the above a(n instance of) `Monad`

```haskell
instance Monad (ST s) where
  return x = STC (\s -> (s, x))
  st >>= f = STC (\s -> let (s', x) = runState st s
                        in runState (f x) s')

runState :: ST s a -> s -> (s, a)
runState (STC f) s = f s

evalState :: ST s a -> s -> a
evalState st s = snd (runState st s)
```

(*exactly* the same code as `returnST` and `bindST` )

# *Lets implement* `keyLabel`

1. Define a `Map Char Int` state-transformer

```
type CharST a = ST (Map Char Int) a
```

2. Modify `next` to take a `Char`

```
charNext :: Char -> CharST Int
charNext c = STC (\m ->
    let
        n  = M.findWithDefault 0 c m    -- label for 'c'
        m' = M.insert c (n+1) m         -- update map
    in
        (m', n)
    )
```

3. Modify `helper` to use `charNext`

```haskell
keyHelperS :: Tree Char -> ST (Tree (Char, Int))
keyHelperS (Leaf c) = do
    n <- charNext c
    return (Leaf (c, n))

keyHelperS (Node l r) = do
    l' <- keyHelperS l
    r' <- keyHelperS r
    return (Tree l' r')

keyLabelS :: Tree Char -> Tree (Char, Int)
keyLabelS t = evalState (keyHelperS t) empty
```

Lets make sure it works!

```
>>> keyLabelS charT
Node
    (Node (Leaf ('a', 0)) (Leaf ('b', 0)))
    (Node (Leaf ('c', 0)) (Leaf ('a', 1)))
```

# *Lets look at the final "state"*

```
>>> (final, t) = runState (keyHelper charT) M.empty
```

The returned `Tree` is

```
>>> t
Node
    (Node (Leaf ('a', 0)) (Leaf ('b', 0)))
    (Node (Leaf ('c', 0)) (Leaf ('a', 1)))
```

and the final `State` is

```
>>> final
fromList [('a',2),('b',1),('c',1)]
```

# *Genericaly Getting and Setting State*

As *State* is "generic"

- i.e. a **type variable** not `Int` or `Map Char Int` or …

It will be convenient to have "generic" `get` and `put` functions

- that *read* and *update* the state

```
-- | `get` leaves state unchanged & returns it as value
get :: ST s s

-- | `set s` changes the state to `s` & returns () as a value
put :: s -> ST s ()
```

# *EXERCISE*

Can you fill in the implementations of `get` and `set` ?

**HINT** Just follow the types…

```haskell
-- | `get` leaves state unchanged & returns it as value
get :: ST s s
get = STC (\oldState -> ???)

-- | `put s` changes the state to `s` & returns () as a value
put :: s -> ST s ()
put s = STC (\oldState -> ???)
```

# *Using `get` and `put` : Global Counter*

We can now implement the plain *global counter* `next` as

```
next :: ST Int Int
next = do
    n <- get      -- save the current counter as 'n'
    put (n+1)     -- update the counter to 'n+1'
    return n      -- return the old counter
```

# *Using* `get` *and* `put` *: Frequency Map*

Lets implement the *char-frequency counter* `charNext` as

```
charNext :: Char -> ST (Map Char Int) Int
charNext c = do
  m    <- get                        -- get current freq-map
  let n = M.findWithDefault 0 c m  -- current freq for c (or 0)
  put (M.insert c (n+1) m)         -- update freq for c
  return n                         -- return current as value
```

# *A State-Transformer Library*

The `Control.Monad.State` module (http://hackage.haskell.org/packages/archive/mtl/latest/doc/html/Control-Monad-State-Lazy.html#g:2)

- defines a State-Transformer like above.
- hides the implementation of the transformer

Clients can **only** use the "public" API

```haskell
-- | Like 'ST s a' but "private", cannot be directly accessed
data State s a

-- | Like the synonyms described above
get       :: State s s
put       :: s -> State s ()
runState  :: State s a -> s -> (a, s)
evalState :: State s a -> s -> a
```

Your homework will give you practice with using these

- to do *imperative functional programming*

# *The IO Monad*

Remember the `IO a` or `Recipe a` type from this lecture (04-haskell-io.html)

- Recipes that return a result of type `a`
- But may also perform some input/output

A number of primitives are provided for building `IO` recipes

```
-- IO is a monad
return  :: a -> IO a
(>>=)   :: IO a -> (a -> IO b) -> IO b
```

Basic actions that can be "chained" via `>>=` etc.

```
getChar :: IO Char
putChar :: Char -> IO ()
```

# *A Recipe to Read a Line from the Keyboard*

```
getLine :: IO String
getLine =  do
  x <- getChar
  if x == '\n' then
      return []
  else do
      xs <- getLine
      return (x:xs)
```

# *IO is a "special case" of the State-Transformer*

The internal state is a representation of the **state of the world**

```
data World -- machine, files, network, internet ...

type IO a  = World -> (World, a)
```

A `Recipe` is a function that

- takes the current `World` as its argument
- returns a value `a` and a modified `World`

The modified `World` reflects any input/output done by the `Recipe`

This is just for understanding, GHC implements `IO` more efficiently! (http://research.microsoft.com/Users/simonpj/papers/marktoberdorf/)

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