Mixing Monads

03-transformers

Monads Can Be Used for Many Things!

- Partial Functions ok/Result
- Global Variables "ST" monad
- Parsing Parsers Comb
- Exceptions
- Test Generation QUICKCHECK
- ~~Concurrency~~
- …

Exception Handling

Recall our expressions with division

```
data Expr
  = Number Int           -- ^ 0,1,2,3,4
  | Plus   Expr Expr     -- ^ e1 + e2
  | Div    Expr Expr     -- ^ e1 / e2
  deriving (Show)
```

We had a **potentially crashing** evaluator

```
eval :: Expr -> Int
eval (Number n)     = n
eval (Plus  e1 e2) = eval e1   +  eval e2
eval (Div   e1 e2) = eval e1 `div` eval e2
                                    0

-- >>> eval (Div (Val 10) (Plus (Number 5) (Number (-5))))
-- Exception: Divide by zero
```

We defined a `Result` *type*

```haskell
data Result a = Ok a | Err String
```

made it a `Monad`

```haskell
instance Monad Result where
  return x      = Ok x
  (Ok v)  >>= f = f v
  (Err s) >>= _ = Err s
```

:: a → Result a

:: Res a → (a → Res b) → Res b

and then we can write

```
eval :: Expr -> Result Int
eval (Number n)    = return n
eval (Plus  e1 e2) = do {n1 <- eval e1; n2 <- eval e2; return (n1   +   n2) }
eval (Div   e1 e2) = do { n1 <- eval e1;
                          n2 <- eval e2;
                          if n2 /= 0
                            then return (n1 `div` n2)
                            else Err ("DBZ: " ++ show e2)
                        }
```

which doesn't crash but returns an `Err`

```
>>> eval (Div (Number 10) (Plus (Number 5) (Number (-5))))
Err "DBZ: Plus (Number 5) (Number (-5))"
```

and when it succeeds it returns an `Ok`

```
>>> eval (Div (Number 10) (Plus (Number 5) (Number (-5))))
Ok 1
```

Generalizing `Result` *to* `Either`

The *standard library* generalizes the `Result` type to `Either`

```
data Result   a = Err String | Ok a

data Either e a = Left e      | Right a
```

- `Err s` becomes `Left s`
- `Ok v` becomes `Right v`
- `Result a` becomes `Either String a`

(But we can data *other* than `String` in the `Left` values)

EXERCISE: Generalizing `Result` *Monad to* `Either` *Monad*

(TRY this at home!)

Lets translate the old `Monad` instance for `Result`

```haskell
instance Monad Result where

  -- return :: a -> Result a
  return x      = Ok x

  -- (>>=) :: Result a -> (a -> Result b) -> Result b
  (Ok v)  >>= f = f v
  (Err s) >>= _ = s
```

RES

into a `Monad` instance for `Either`

```
instance Monad (Either e) where
  -- return :: a -> Either e a
  return x          = ~~???~~ Right x          // ok -> Right

  -- (>>=) :: Either e a -> (a -> Either e b) -> Either e b
  (Right v) >>= f = ~~???~~ f v
  (Left  s) >>= _ = ~~???~~ Left s
```

QUIZ

We can rewrite `eval` to return an `Either`

left

le "right

```
eval :: Expr -> Either Expr Int
eval (Number n)    = return n
eval (Plus  e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        return (n1+n2)
eval (Div   e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        if n2 /= 0
                          then return (n1 `div` n2)
                          else Left e2
```

What does `quiz` evaluate to?

quiz :: Either Expr Int

```
quiz = eval (Div (Val 10) (Plus (Number 5) (Number (-5))))
```

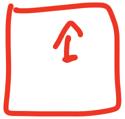
A. `Err "DBZ: Plus (Number 5) (Number (-5))"`

[✓ **B.**] `Left "DBZ: Plus (Number 5) (Number (-5))"`

C. Run-time Exception

D. `Plus (Number 5) (Number (-5))`

E. `Left (Plus (Number 5) (Number (-5)))` ✓



exn

`Either` is an Exception Monad!

What can you do with exceptions?

1. `throwError` an exception (with some value) …
2. `catchError` an exception (and use its value) …

① throw new Exn...

② try {....}
catch (e) {...} handle
finally {...}

1. `throw`ing an Exception

We can simply define

```
throw :: e -> Either e a
throw exn = Left exn
```

and now *voila*

```
eval :: Expr -> Either Expr Int
eval (Number n)    = return n
eval (Plus  e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        return (n1 + n2)
eval (Div   e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        if n2 /= 0
                          then return (n1 `div` n2)
                          else throw e2
```

Exactly the same evaluator

- Result is a `Left` ==> an *exception* came all the way to the top.
- `Either` monad ensures the “exception” shoots to the top!

```
>>> eval (Div (Numer 10) (Plus (Number 5) (Number (-5))))
Left (Minus (Number 5) (Number 5))
```

No further evaluation happens after a `throw` because ???

`catch`*ing an exception*

How to *catch* an exception?

Lets change our `Expr` type to

```
data Expr
  = Number  Int             -- ^ 0,1,2,3,4
  | Plus    Expr Expr       -- ^ e1 + e2
  | Try     Expr Int
  deriving (Show)
```

Informally, `try e n` evaluates to `e` but

- if `e` is undefined due to *divide-by-zero*

- then evaluate to `n`

```
eval :: Expr -> Either Expr Int
eval (Number n)    = return n
eval (Plus  e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        return (n1+n2)
eval (Div   e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        if n2 /= 0
                          then return (n1 `div` n2)
                          else throw e2
eval (Try e n)     = catch (eval e) (\_ -> return n)
```

tryCatch
e
(\exn → ...)

try {...} (exn) {...}

QUIZ

What should the *type* of `catch` be?

A. ~~`Either e a -> (a -> Either e b) -> Either e b`~~

B. ~~`Either e a -> (e -> Either e b) -> Either e b`~~

C. `Either e a -> (e -> Either e a) -> Either e a`

D. ~~`Either e a -> Either e a -> Either e a`~~

E. ~~`Either e a -> Either e b -> Either e b`~~

Implementing `catch`

Lets implement the `catch` function!

```
catch :: Either e a -> (e -> Either e a) -> Either e a
catch (Left  e) handler = ???
catch (Right a) handler = ???
```

QUIZ

```
catch :: Either e a -> (e -> Either e a) -> Either e a
catch (Left  e) handle  = ???
catch (Right a) handler = ???

eval :: Expr -> Either Expr Int
eval (Number n)    = return n
eval (Plus  e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        return (n1+n2)
eval (Div   e1 e2) = do n1 <- eval e1
                        n2 <- eval e2
                        if n2 /= 0
                          then return (n1 `div` n2)
          Def             else throw e2
eval (~~Try~~ e n)     = try catch (eval e) (\_ -> return n)

e1  = Div (Number 10) (Plus (Number 5) (Number (-5)))
e1' = ~~Try~~ e1 7
      Def

quiz = eval (Try e1 7)
```

What does `quiz` evaluate to?

A. `Right 7` ✓

B. `Left 7` x because "return 7 → Right 7"

C. `Right 0`

D. `Left 0`

E. `Left (Plus (Number 5) (Number (-5)))`

`Either` *is an* ***Exception*** *Monad!*

1. `throw` an exception (with some value) …

2. `catch` an exception (and use its value) …

```
throw :: e -> Either e a
throw e = Left e

catch :: Either e a -> (e -> Either e a) -> Either e a
catch (Left  e) handle = handle e
catch (Right e) _      = Right  e
```

Monads Can Be Used for Many Things!

- Partial Functions ✓
- Global State ✓

ST

- Parsing ✓
- Exceptions ✓
- Test Generation
- Concurrency
- …

Either

… but what if I want *Exceptions* **and** *Global State* ?

Mixing Monads

What if I want *Exceptions* **and** *Global State* ?

Profiling with the ST Monad

Lets implement a *profiling* monad that counts the number of operations

```
-- A State-Transformer with a "global" `Int` counter
type Profile a = State Int a
```

We can write a `runProfile` that

- executes the transformer from `0`
- and renders the result

```
runProfile :: (Show a) => Profile a -> String
runProfile st = showValCount (runState st 0)
```

(v, c)

```
showValCount :: (Show v, Show c) => (v, c) -> String
showValCount (val, count) = "value: " ++ show val ++ ", count: " ++ show count
```

A function to *increment* the counter

```
count :: Profile ()
count = do
  n <- get
  put (n+1)
```

A Profiling Evaluator

We can use `count` to write a *profiling* evaluator

```
evalProf :: Expr -> Profile Int
evalProf = eval
  where
    eval (Number n)    = return n
    eval (Plus  e1 e2) = do n1 <- eval e1
                            n2 <- eval e2
                            count
                            return (n1+n2)
    eval (Div   e1 e2) = do n1 <- eval e1
                            n2 <- eval e2
                            count
                            return (n1 `div` n2)
```

And now, as there are *two* operations, we get

```
>>> e1
Div (Number 10) (Plus (Number 5) (Number 5))

>>> runProfile (evalProf e1)
"value: 1, count: 2"
```

But what about Divide-by-Zero?

Bad things happen…

```
>>> e2
Div (Number 10) (Plus (Number 5) (Number (-5)))

>>> runProfile (evalProf e2)
*** Exception: divide by zero
"value:
```

Problem: How to get *global state* AND *exception handling* ?